# TRANSIENT TEMPERATURE DISTRIBUTION IN ROTARY REGENERATOR HEAT EXCHANGER

### ITSURO SAKAI AND YOSHIHIRO KAWAGUCHI

Office of Truck & Bus Development & Engineering, Mitsubishi Motors Corporation, 21-4 Shimomaruko, Ohta-ku, Tokyo, 145, Japan

### ABSTRACT

The purpose of this study is to analyse dynamic response of rotary regenerator for dynamic simulation of automotive gas turbine using a simplified method. Exit temperatures of working fluids in rotary regenerator depend on time, position of core, inlet temperature of working fluid and initial temperature of core. We tried to simplify the calculation model of rotary regenerator, as a counter flow heat exchanger and show that they give good results compared to the available analytical solution.

KEY WORDS Rotary regenerator Ceramic honeycomb Gas turbine Dynamic response

# NOMENCLATURE

A	heat transfer area of a		unit $(= nAh/C)$
	divided each core	ť	time (sec)
	element $(m^2)$	t	time (dimensionless)
$C_{a}$	heat capacity of working		$(=C_a \cdot t'/C_M \text{ or }$
-	air $(W/K)$		$C_{a}t'/C_{M}$
$C_{a}$	heat capacity of working	$\Delta t'$	cycle time (sec)
5	gas (W/K)	$\Theta_k (k=0 \sim n-1)$	temperature of each core
$C_{M}$	heat capacity of a		element
	divided each core	$\theta_{a,k} (k=0 \sim n)$	temperature of working
	element (J/K)		air (K)
h <sub>a</sub>	heat transfer coefficient	$\theta_{g,k} \ (k=0 \sim n)$	temperature of working
-	between core and air		gas (K)
	$(W/m^2K)$	E	heat transfer
ha	heat transfer coefficient		effectiveness
5	between core and gas		$(=(\theta_{g,0}-\theta_{g,n})/$
	$(W/m^2K)$		$(\theta_{q,0} - \theta_{a,0}))$
n	number core element	Ω	cycle time
	divisions		$(=(C_a\cdot\Delta t'/(n\cdot C_M))$ or
NTU	number of heat transfer		$(C_{a} \cdot \Delta t' / (n \cdot C_{M}))$

### INTRODUCTION

In the high efficiency automotive gas turbine, the regenerative cycle with a heat exchanger is used. For this cycle, the exhaust thermal energy is used by a rotary regenerator to heat the

0961-5539/93/020173-10\$2.00 © 1993 Pineridge Press Ltd Received June 1992 Revised December 1992



Figure 1 Gas turbine cycle with heat exchanger

compressed air. This rotary regenerator works between the high temperature exhaust gas passage and the low temperature compressed air as shown in *Figure 1*.

Compactness is the main required characteristic, for this type of heat exchanger. However, since the heat capacity of the rotary generator is comparatively large, a time lag, which occurs in the regenerator outlet air, gas temperature and regenerator temperature has to be considered. Because of large fluctuations of vehicle speed and load, this time-lag considerably affects the engine performance and the thermal stress in the regenerator itself. Therefore, it is very important to determine the transient response quantitatively.

When the component characteristics of the compressor, turbine, combustor, heat exchanger, etc., and their leakage loss, pressure loss, mechanical loss, and heat loss are given, the steady state operation point and characteristics of the gas turbine can be obtained by flow balance, pressure balance, and energy balance. However, the transient response becomes a problem in the automotive gas turbine. It is determined by the simulation of dynamic response, in which simultaneous differential equations are solved after modelling the rotating system, drive line, vehicle, and control system.

In this case, since the above-mentioned transient response which arises from the heat capacity of the regenerator core in the rotary generator greatly affects the transient characteristics of the automotive gas turbine, especially the fuel consumption when running in an urban area, the exact modelling is required in the dynamic characteristic simulation. The conventional transient response model of the rotary regenerator is treated as the first-order time-lag system considering the effect of the air flow. In the case of automobiles, however, the temperature distribution of the regenerator core varies according to the operating conditions, i.e. cold starting, acceleration from idling, quick acceleration/deceleration, slow acceleration/deceleration, etc. Without considering such effects, the conventional model is not good for the dynamic characteristic simulation.

The purpose of this study is to determine the influence of the initial temperature distribution of the rotary regenerator core on the transient response, and at the same time to make the modelling of the above heat exchanger more accurate.

# ANALYSIS

# Analytical model

Regarding the temperature distribution of the rotary regenerator core during operation (Figure 2) the temperature distribution of the core across the thickness is large, while the circumferential distribution is relatively small<sup>1</sup>. The heat exchanger characteristics approach the counter flow type as the rotational core speed is increased.

Figure 3a shows the analytical model corresponding to Figure 3b. We assume that the thickness dimension of the core is divided into n elements and the temperature distribution of each element is uniform. The heat capacities of the working fluids, air and gas, shall be  $C_a$  and  $C_a$  (W/K) respectively.

### **Basic** equations

First, the heat balance for the core element 0 is given below:

$$C_M \frac{\mathrm{d}\Theta_0}{\mathrm{d}t'} = (\theta_{a,0} - \Theta_0)Ah_a + (\theta_{g,n-1} - \Theta_0)Ah_g \tag{1}$$

$$C_a(\theta_{a,1} - \theta_{a,0}) = (\Theta_0 - \theta_{a,0})Ah_a$$
<sup>(2)</sup>

$$C_g(\theta_{g,n} - \theta_{g,n-1}) = (\Theta_0 - \theta_{g,n-1})Ah_g$$
(3)



Figure 3 Rotary regenerator models

 $\theta_{g,0}$ 

In the same way, for elements  $1, 2, \ldots, n-1$ ,

. \_

•

$$C_M \frac{\mathrm{d}\Theta_1}{\mathrm{d}t'} = (\theta_{a,1} - \Theta_1)Ah_a + (\theta_{g,n-2} - \Theta_1)Ah_g \tag{4}$$

$$C_a(\theta_{a,2} - \theta_{a,1}) = (\Theta_1 - \theta_{a,1})Ah_a$$
<sup>(5)</sup>

$$C_g(\theta_{g,n-1} - \theta_{g,n-2}) = (\Theta_1 - \theta_{g,n-2})Ah_g$$
(6)

$$C_{\mathcal{M}} \frac{\mathrm{d}\Theta_{n-1}}{\mathrm{d}t'} = (\theta_{a,n-1} - \Theta_{n-1})Ah_a + (\theta_{g,0} - \Theta_{n-1})Ah_g \tag{7}$$

$$C_a(\theta_{a,n} - \theta_{a,n-1}) = (\Theta_{n-1} - \theta_{a,n-1})Ah_a$$
(8)

$$C_{g}(\theta_{g,1} - \theta_{g,0}) = (\Theta_{n-1} - \theta_{g,0})Ah_{g}$$
(9)

Here,

$$(C_a/C_M) \cdot dt' = dt \quad (\text{dimensionless}) \tag{10}$$

$$C_g/C_a = \beta \tag{11}$$

$$N_a = Ah_a/C_a \tag{12}$$

$$N_g = Ah_g/C_g \tag{13}$$

(Note: for the whole core,  $NTU_a = nN_a = nAh_a/C_a$ ,  $NTU_g = nN_g = nAh_g/C_g$ .) By substituting (10)-(13) into (2), (3), (5), (6), (8) and (9),

$$\theta_{a,1} = (1 - N_a)\theta_{a,0} + N_a\Theta_0 \tag{14}$$

$$\sigma_{a,2} = (1 - N_a)\sigma_{a,1} + N_a\Theta_1$$
  
=  $(1 - N_a)[(1 - N_a)\theta_{a,0} + N_a\Theta_0] + N_a\Theta_1$   
=  $(1 - N_a)^2\theta_{a,0} + N_a(1 - N_a)\Theta_0 + N_a\Theta_1$  (15)

$$\theta_{a,n} = (1 - N_a)^n \theta_{a,0} + N_a (1 - N_a)^{n-1} \Theta_0 + N_a (1 - N_a)^{n-2} \Theta_1 + \dots + N_a (1 - N_a) \Theta_{n-2} + N_a \Theta_{n-1}$$
(16)

In the same way,

•

•

$$\theta_{g,1} = (1 - N_g)\theta_{g,0} + N_g\Theta_{n-1}$$

$$\theta_{g,2} = (1 - N_g)\theta_{g,1} + N_g\Theta_{n-2}$$
(17)

$$= (1 - N_g)[(1 - N_g)\theta_{g,0} + N_g\Theta_{n-1}] + N_g\Theta_{n-2}$$
  
=  $(1 - N_g)^2\theta_{g,0} + N_g(1 - N_g)\Theta_{n-1} + N_g\Theta_{n-2}$  (18)

$$\theta_{g,n} = (1 - N_g)^n \theta_{g,0} + N_g (1 - N_g)^{n-1} \Theta_{n-1} + N_g (1 - N_g)^{n-2} \Theta_{n-2} + \dots + N_g (1 - N_g) \Theta_1 + N_g \Theta_0$$
(19)

By substituting (14)-(19) into  $\Theta_0, \Theta_1, \ldots, \Theta_{n-1}$  shown in (1), (4) and (7), we get:

•

$$\frac{d\Theta_{0}}{dt} = -(N_{a} + N_{g}\beta)\Theta_{0} + N_{g}^{2}\beta[\Theta_{1} + (1 - N_{g})\Theta_{2} + (1 - N_{g})^{2}\Theta_{3} + \cdots + (1 - N_{g})^{n-3}\Theta_{n-2} + (1 - N_{g})^{n-2}\Theta_{n-1}] + \theta_{a,0}N_{a} + N_{g}\beta(1 - N_{g})^{n-1}\theta_{g,0}$$
(20)  
$$\frac{d\Theta_{1}}{dt} = N_{g}^{2}\Theta_{0} - (N_{a} + N_{g}\beta)\Theta_{1} + N_{g}^{2}\beta[\Theta_{2} + (1 - N_{g})\Theta_{3} + (1 - N_{g})^{2}\Theta_{4} + \cdots + (1 - N_{g})^{n-4}\Theta_{n-2} + (1 - N_{g})^{n-3}\Theta_{n-1}] + N_{a}(1 - N_{a})\theta_{a,0} + N_{g}\beta(1 - N_{g})^{n-2}\theta_{g,0}$$
(21)

$$\frac{\mathrm{d}\Theta_{n-1}}{\mathrm{d}t} = N_a^2 [(1 - N_a)^{n-2}\Theta_0 + (1 - N_a)^{n-3}\Theta_1 + (1 - N_a)^{n-4}\Theta_2 + \dots + \Theta_{n-2}] - (N_a + N_g\beta)\Theta_{n-1} + N_a(1 - N_a)^{n-1}\theta_{a,0} + N_g\beta\theta_{g,0}$$
(22)

In (14)-(22), by selecting the number of elements, so as to satisfy  $N_a = N_g = 1$  when  $NTU_a = NTU_g$  is set, the equations will be as follows:

$$\frac{d\Theta_{0}}{dt} + (1+\beta)\Theta_{0} - \beta\Theta_{1} = \theta_{a,0}$$

$$-\Theta_{0} + \frac{d\Theta_{1}}{dt} + (1+\beta)\Theta_{1} - \beta\Theta_{2} = 0$$

$$-\Theta_{1} + \frac{d\Theta_{2}}{dt} + (1+\beta)\Theta_{2} - \beta\Theta_{3} = 0$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$-\Theta_{n-2} + \frac{d\Theta_{n-1}}{dt} + (1+\beta)\Theta_{n-1} = \beta\theta_{g,0}$$
(23)

When d/dt = P and  $\beta = 1$  are set in the above equation, by expressing each initial value of  $\Theta_0, \Theta_1, \ldots, \Theta_{n-1}$  in (23)  $\Theta_{0,0}, \Theta_{0,1}, \ldots, \Theta_{0,n-1}$ , they are expressed as follows:

As shown in (24), they form a system of simultaneous ordinary differential equations. On the other hand, from (14)-(19),  $\theta_{a,1}, \theta_{a,2}, \ldots, \theta_{a,n}$  and  $\theta_{g,1}, \theta_{g,2}, \ldots, \theta_{g,n}$  becomes:

$$\begin{array}{cccc} \theta_{a,1} = \Theta_{0} & \theta_{g,1} = \Theta_{n-1} \\ \theta_{a,2} = \Theta_{1} & \theta_{g,2} = \Theta_{n-2} \\ & & & \\ & & & \\ & & & \\ & & & \\ \theta_{a,n} = \Theta_{n-1} & \theta_{g,n} = \Theta_{0} \end{array} \right\}$$

$$(25)$$

### Calculation method

Before obtaining  $\Theta_0, \Theta_1, \ldots, \Theta_{n-1}$  from equation (24), it is necessary to obtain eigenvalues  $P_k$  ( $k = 0 \sim n-1$ ) first which satisfy 'left side P's matrix of equation (24) = 0'. This form of matrix is the same as the one to determine the eigenvalues of the vibration system connected with strings in which tensions are applied between n pieces of mass points, and n pieces of eigenvalues are expressed as<sup>2</sup>:

$$P_k = -2[1 - \cos((k+1)/(n+2))] \qquad (k = 0, 1, 2, \dots, n-1)$$
(26)

By expanding the left side P's of matrix of (24), it becomes nth degree polynomial of P, this is expressed by  $\Delta(P)$ :

$$\Delta(P) = (P - P_0) \cdot (P - P_1) \cdots (P - P_{n-1})$$
(27)

For example,  $\Theta_0$  is solved as follows using Laplace transform:

$$\Theta_{0}(t) = \Theta_{0,0} \left[ \frac{A_{1,1}(P_{0})}{\Delta'(P_{0})} \exp(P_{0} \cdot t) + \frac{A_{1,1}(P_{1})}{\Delta'(P_{1})} \exp(P_{1} \cdot t) + \dots + \frac{A_{1,1}(P_{n-1})}{\Delta'(P_{n-1})} \exp(P_{n-1} \cdot t) \right] \\ + \Theta_{0,1} \left[ \frac{A_{2,1}(P_{0})}{\Delta'(P_{0})} \exp(P_{0} \cdot t) + \frac{A_{2,1}(P_{1})}{\Delta'(P_{1})} \exp(P_{1} \cdot t) + \dots + \frac{A_{2,1}(P_{n-1})}{\Delta'(P_{n-1})} \exp(P_{n-1} \cdot t) \right]$$

$$+ \Theta_{0,n-1} \left[ \frac{A_{n,1}(P_0)}{\Delta'(P_0)} \exp(P_0 \cdot t) + \frac{A_{n,1}(P_1)}{\Delta'(P_1)} \exp(P_1 \cdot t) + \dots + \frac{A_{n,1}(P_{n-1})}{\Delta'(P_{n-1})} \exp(P_{n-1} \cdot t) \right] + \theta_{a,1} \left[ -\frac{1}{P_0} \frac{A_{1,1}(P_0)}{\Delta'(P_0)} \left(1 - \exp(P_0 \cdot t)\right) - \frac{1}{P_1} \frac{A_{1,1}(P_1)}{\Delta'(P_1)} \left(1 - \exp(P_1 \cdot t)\right) - \dots \right] - \frac{1}{P_{n-1}} \frac{A_{1,1}(P_{n-1})}{\Delta'(P_{n-1})} \left(1 - \exp(P_{n-1} \cdot t)\right) \right] + \theta_{g,n} \left[ -\frac{1}{P_0} \frac{A_{n,1}(P_0)}{\Delta'(P_0)} \left(1 - \exp(P_0 \cdot t) - \frac{1}{P_1} \frac{A_{n,1}(P_1)}{\Delta'(P_1)} \left(1 - \exp(P_1 \cdot t)\right) - \dots \right] - \frac{1}{P_{n-1}} \frac{A_{n,1}(P_{n-1})}{\Delta'(P_{n-1})} \left(1 - \exp(P_{n-1} \cdot t)\right) \right]$$
(28)

where

$$A_{i,j}(i = 1 \sim n, j = 1 \sim n)$$
 is the cofactor of P's matrix of (24)  
$$\Delta'(P) = \frac{d}{dP} \Delta'(P)$$

In the same way,  $\Theta_1(t), \Theta_2(t), \ldots, \Theta_{n-1}(t)$  can be obtained. From (25), the outlet temperature of the air passing through the regenerative heat exchanger  $\theta_{a,n}(t)$  will be:

$$\theta_{a,n}(t) = \Theta_{n-1}(t) \tag{29}$$

# CALCULATION EXAMPLE AND CONSIDERATION

Figure 4 shows the relationship between the heat transfer effectiveness E considering the circumferential temperature distribution of regenerator core and the cycle time  $\Omega$ . The difference between the effectiveness for the practical range of  $\Omega = 0.5-1.5$  and that for  $\Omega = 0$  (rotational speed  $= \infty$ ) is within 0.4% in case of  $NTU_a = NTU_g = 20$ . This analysis is equivalent to the counter flow type heat exchanger (internal resistance = 0) in which case  $\Omega = 0$ .

Here, the calculation is carried out for the cases of  $NTU_a = NTU_g = 10$ , that is, n = 5, and  $NTU_a = NTU_g = 20$ , that is, n = 10.

According to (26), the eigenvalues for n = 5 and 10 is calculated as follows:

(n=5)	(n = 10)		
$P_0 = -0.26791924$	$P_0 = -0.08101405$	$P_5 = -2.2846297$	
$P_1 = -1$	$P_1 = -0.31749293$	$P_6 = -2.8308300$	(20)
$P_2 = -2$	$P_2 = -0.69027853$	$P_7 = -3.3097215$	(30)
$P_3 = -3$	$P_3 = -1.16916997$	$P_8 = -3.6825071$	
$P_4 = -3.732050801$	$P_4 = -1.71537032$	$P_9 = -3.9189899$	



Figure 4 Effectiveness of rotary regenerator



Figure 5 Transient temperature changes in the regenerator core and working fluid (air)

Using these eigenvalues, the outlet temperature of the working air for n = 5 is expressed as follows.

$$\theta_{a,5} = 0.31100299(1 - \exp(P_0 \cdot t)) + 0.25(1 - \exp(P_1 \cdot t)) + 0.166665(1 - \exp(P_2 \cdot t)) + 0.083333(1 - \exp(P_3 \cdot t)) + 0.022329(1 - \exp(P_4 \cdot t))$$
(31)

### Influence of regenerator core circumferential temperature distribution on effectiveness

Figure 5 shows the temperature change of the regenerator core/working air when the initial temperature of the regenerator core is 0, between this analysis (shown in Figure 3a) and analytical solution (shown in Figure 3b) of Reference 1.

The case of  $\Omega = 0$  of *Figure 4*, exactly corresponds to counter flow type heat exchanger performance. It is clearly seen that the air temperature change in the present analysis without considering the circumferential temperature distribution of the regenerator core corresponds well to that with considering it in the analytical solution<sup>1</sup> and in the periodic steady state, the effectiveness of present analysis agrees with that of analytical solution.

Since this physical phenomenon is a continuous unsteady state, it is necessary to calculate it every half cycle, i.e. cooling cycle and heating cycle theoretically. In this analytical counter flow model, to keep the same values of effectiveness of rotary regenerator in steady state, we take half the values of *NTU* in rotary regenerator (see Appendix).

# Influence of NTU value on effectiveness (Cold starting, $NTU_a = NTU_g = 10, 20$ )

Figure 6 shows the working air temperature change in the case of  $NTU_a = NTU_g = 20$ . It is natural that the effectiveness in the steady state should be higher than that for  $NTU_a = NTU_g = 10$  shown in Figure 5. This is also seen from effectiveness  $E = NTU_0/(1 + NTU_0)$  in the case of  $\Omega = 0$ .  $NTU_0$  means overall NTU ( $NTU_0$  is defined in the Appendix).

### Transient characteristics $(NTU_a = NTU_g = 10)$

Figure 7 shows the working air temperature change in the transient state. This starts with the regenerator core temperature  $\Theta_k$  (k = 0-4) = 0, inlet air temperature  $\theta_{a,0} = 0$ , and inlet gas temperature  $\theta_{g,0} = 1.0$ . In the next step, the inlet gas temperature falls to  $\theta_{g,0} = 0.5$  in stepwise manner after becoming steady (( $C_a/C_M$ )t' = 15), then the inlet gas temperature attains to  $\theta_{g,0} = 1.0$  after ( $C_a/C_M$ )t' = 20.



Figure 6 Transient temperature changes of the working fluid (air)



Figure 7 Transient temperature changes of the working fluid (air)

From the above, it can be seen that it is important to keep the regenerator core temperature as high as possible and to prevent from cooling in automotive gas turbine.

From (26), it is seen that in case of large NTU value, the minimum eigenvalue of them becomes a small value; on the other hand, a small eigenvalue is the same as a large time constant. Therefore the transient state included a small eigenvalue will be finished with a large time constant. It agrees with engineering experience.

#### Regenerator temperature

As is evident from the above, the regenerator core temperature does not need to be handled as a heat storage distributed system but can be approximated as a heat storage lumped system only when the *NTU* of the whole system is as small as  $NTU_a = NTU_g = 1$ , i.e. n = 1. Therefore, it becomes possible to handle the regenerator core temperature as a physical phenomenon of first-order time-lag only when n = 1.

### CONCLUSIONS

As shown in *Figure 2*, when the regenerator core circumferential temperature distribution can be relatively small, the simplified calculation model of the rotary generator shows that the effectiveness is the same as the counter flow heat exchanger model if the core is divided into elements whose thickness is in accordance with the *NTU* value. We can conclude the following from this study:

- (1) The effectiveness of the rotary regenerator becomes very close to the analytical solution.
- (2) The transient response can be said to be good enough for practical use by neglecting the temperature variation at each core rotation.

The outlet temperatures of the air and the gas are directly expressed with one equation respectively, by giving the initial temperatures of each core element and the inlet temperature on the gas and air sides. This calculation is very simple compared with the analytical solution<sup>1</sup>.

# ACKNOWLEDGEMENTS

We thank Professor Kunio Hijakata of Tokyo Institute of Technology for his advice. Also the authors wish to thank H. Ogita for his kind help in analysing this investigation.

### REFERENCES

1 Kawasaki, K., Matsuhisa, T., Sakai, I. and Hijikata, K. Heat transfer characteristics of rotating ceramic regenerator, ASME Paper, No. 91-GT-371 (1991)

2 Japobsen, L. S. and Ayre, R. S. Engineering of Vibration with Application to Structures and Machinery, McGraw-Hill, New York (1959)

### APPENDIX

The relation between the heat exchange schematic model of rotary regenerator type and counter flow type is shown as follows.



Figure 8 Characteristics of heat exchanger's effectiveness

In case of the following conditions,

 $NTU = NTU_a = NTU_a, \qquad \beta = 1 \ (= C_a/C_a)$ 

Effectiveness of rotary regenerator and counter flow type heat exchanger becomes:

$$E_R = \frac{NTU_0}{1 + NTU_0} = \frac{NTU}{2 + NTU} \qquad (\Omega \le 1.5, \text{ see Figure 4})$$

where

$$\frac{1}{NTU_0} = \frac{1}{NTU_a} + \frac{1}{NTU_g} = \frac{1}{1/2NTU}$$

For counter flow type

$$E_c = \frac{NTU}{1 + NTU}$$

In this paper, the calculation result of rotary regenerator dynamic simulation will give the same effectiveness between rotary regenerator and counter flow type in steady state, if NTU value of counter flow type corresponds to half the value of rotary regenerator NTU. Figure 8 shows the  $E_R$ ,  $E_C$  vs. NTU.